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Prediction in stratified gas–liquid co-current flow in horizontal pipelines

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Abstract—The predictive performance of existing models based on momentum balances has been shown to be generally unsatisfactory. An iterative procedure was developed for the prediction of pressure drop and holdup that incorporated new relationships for the interfacial and liquid friction factors in the solution of the phase momentum balance equations for two phase horizontal co-current flow. The method adequately predicted data for the film plus droplet, annular roll wave and stratified type regimes. Successful performance was achieved regardless of fluid properties or pipe diameter. For gas flow rates, where non-uniform stratified flow with an interfacial level gradient occurred, this method of prediction was inaccurate and open channel flow theory recommended. Thus the model did not apply to the intermittent or non-uniform flow regimes.

1. INTRODUCTION

Despite numerous theoretical and experimental investigations into gas liquid pipe flow, no general models are available that reliably predict frictional pressure loss and liquid holdup in horizontal flow. Spedding *et al.* [1–4] have reviewed the prediction performance of various holdups models, while Spedding *et al.* [5–9] have reported on pressure loss prediction methods.

However, as Spence and Spedding [5], Xiao *et al.* [10] and Spedding *et al.* [2, 4] have shown, all successful models were of limited usefulness, being among other things flow regime dependent. Better agreement between experiment and theory was found when a phenomenological approach was used to model gas liquid flow.

A useful approach has been to model two-phase stratified flow using one-dimensional momentum balances over each phase. The pressure loss came from the resistance between the pipe wall and the phases and from interfacial effects.

Figure 1 illustrates the principal geometric parameters and shear forces developed in co-current smooth stratified flow. A one-dimensional momentum balance across each phase produced

$$-A_L(dP/dL)_L - \tau_{WL}\cdot S_L + \tau_i\cdot S_i - \rho_L A_L g\sin\alpha = 0 \tag{1}$$

$$-A_G(dP/dL)_G - \tau_{WG}\cdot S_G - \tau_i\cdot S_i - \rho_G A_G g\sin\alpha = 0. \tag{2}$$

The shear stresses are defined

$$\tau_{WL} = f_L\frac{\rho_L V_L^2}{2} \tag{3}$$

$$\tau_{WG} = f_G\frac{\rho_G V_G^2}{2} \tag{4}$$

$$\tau_i = f_i\rho_G\frac{(V_G - V_L)^2}{2}. \tag{5}$$

The parameters A_L, A_G, S_L, S_G and S_i, are geometric functions of the dimensionless liquid height, h_L/D, and are detailed in the Appendix.

Figure 2 presents the result graphically for the liquid gas turbulent–turbulent case. Single-valued solutions to the overall relation were obtained only if $Y \geqslant -3.8$ according to Landman [11].

For uniform stratified flow in a horizontal pipeline $(dP/dL)_L = (dP/dL)_f = (dP/dL)_G$ and $\alpha = 0$. Therefore by adding equations (1) and (2) an expression for the liquid–wall, shear stress is obtained.

$$\tau_{WL} = \frac{-(dP/dL)_f A - \tau_{WG}S_G}{S_L}. \tag{6}$$

NOMENCLATURE

A cross-sectional area
D pipe diameter
f friction factor
g acceleration due to gravity
h film height
h_L^+ dimensionless liquid film height of equation (10)
k_i variable, equation (23)
$(dP/dL)_L$ pressure loss per unit length of pipeline in liquid phase
$(dP/dL)_G$ pressure loss per unit length of pipeline in gas phase
$\bar{R}$ hold-up
Re Reynolds number
S perimeter
V velocity
X Lockhart–Martinelli [26] parameter $[(dP/dL)_L/(dP/dL)_G]^{1/2}$
Y Lockhart–Martinelli [26] parameter $(((\rho_L-\rho_G)g\sin\alpha)/(dP/dL)_G)$.

Greek symbols
α angle of inclination of the pipe with respect to the horizontal, with angles downward in the direction of the flow assumed to be positive
β input volumetric ratio
ρ density
τ_C characteristic shear stress of equation (11)
τ_W wall shear stress
ν kinematic viscosity.

Subscripts
f friction
G gas phase
i interface
L liquid phase
S superficial.

Superscript
$\hat{}$ dimensionless parameter.

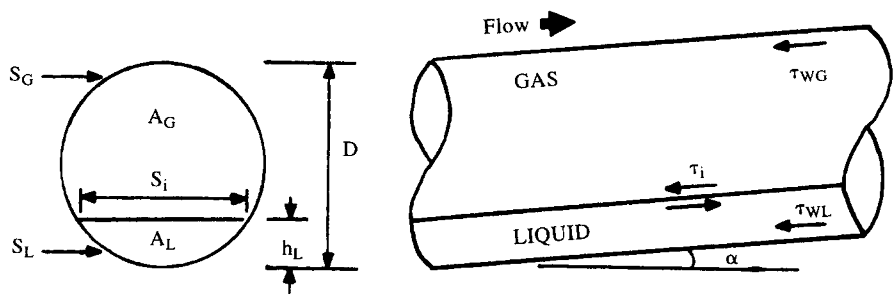

Fig. 1. Smooth stratified flow in a pipeline.

Spedding and Hand [7, 9, 12] have shown that the relation assigned to the interfacial shear was of crucial importance and have discussed and evaluated the various approaches which have been used. Some of these have proved to be useful and will be used in this work. Others will be mentioned as they are useful to the discussion.

Agrawal *et al.* [13] assumed the gas–liquid interface to be hydrodynamically smooth with f_i calculated using the Ellis and Gay [14] interfacial friction factor defined by equation (7).

$$f_i = 1.293 Re_G^{-0.57}. \quad (7)$$

Cheremisinoff and Davis [15] proposed a model for turbulent liquid, turbulent gas stratified flow in pipelines utilizing the interfacial friction factor relationships proposed by Cohen and Hanratty [16] and Miya *et al.* [17] for two-dimensional small amplitude waves and roll waves as defined by equations (8) and (9), respectively.

$$f_i = 0.0142 \quad (8)$$

$$f_i = 0.008 + 2\times10^{-5} Re_L. \quad (9)$$

Neither of the models by Agrawal *et al.* [13] and Cheremisinoff and Davis [15] predicted satisfactorily according to Spedding and Hand [7, 9].

Andritsos [18] conducted experiments in horizontal pipelines having diameters of 0.0252 m and 0.09525 m with liquid viscosities ranging from 1 to 80 cp. An iterative solution to the phase momentum balance equations (i.e. equations (1) and (2)) was proposed with new equations for the liquid wall shear stress (τ_{WL}) and interfacial friction factor (f_i). He suggested that T_{WL} could be predicted using a correlation for dimensionless liquid film height (h_L^+) and the liquid phase Reynolds number which was capable of accounting for changes in the liquid phase velocity profile caused by gas drag at the gas–liquid interface.

$$h_L^+ = [(1.082 Re_L^{0.5})^5 + [0.098 Re_L^{0.85}/(1-h_L/D)^{0.5}]^5]^{0.2} \quad (10)$$

$$\tau_C = \rho_L \left[\frac{h_L^+ \nu_L}{D}\right]^2 \left[\frac{h_L}{D}\right]^{-2} \quad (11)$$

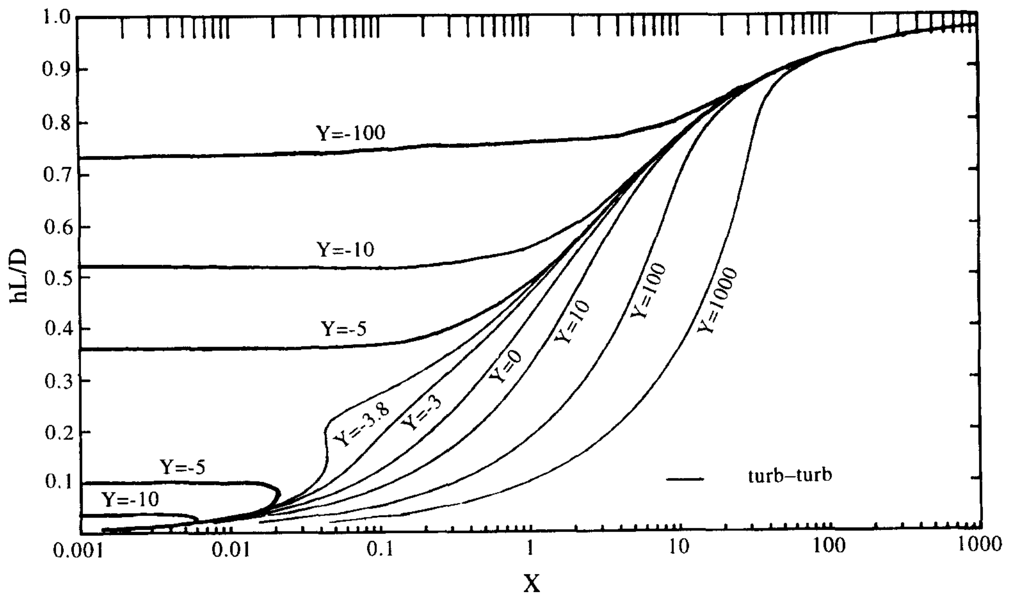

Fig. 2. The Taitel and Dukler [25] relation between liquid level and the Lockhart–Martinelli parameter (X) for turbulent–turbulent flow in a 0.0455 m i.d. pipe. The pipeline is inclined upwards for negative values of the parameter *Y*.

$$\tau_{WL} = \frac{3\tau_C - \tau_i}{2(1-h_L/D)}. \quad (12)$$

A novel correlation was proposed for f_i, when the gas superficial velocity was in excess of the velocity necessary to initiate large amplitude roll waves, i.e. $(V_{SG})_t$, with f_i defined by equations (13) and (14) for smooth stratified/stratified ripple flow and stratified roll wave flow, respectively.

$$\frac{f_i}{f_G} = 1 \quad \text{for } V_{SG} < (V_{SG})_t \quad (13)$$

$$\frac{f_i}{f_G} = 1 + 15\left[\frac{h_L}{D}\right]^{0.5}\left[\frac{V_{SG}}{(V_{SG})_t} - 1\right]. \quad (14)$$

Spedding and Hand [7] showed that the model predicted pressure loss satisfactorily in the stratified long and short roll wave and droplet regimes

$$f_{L,G} = 16 Re_{L,G}^{-1} \quad Re_{L,G} < 2100 \quad (15)$$

$$f_{L,G} = 0.046 Re_{L,G}^{-0.2} \quad Re_{L,G} > 4000. \quad (16)$$

Kowalski [19] demonstrated that measurements of f_G were accurately predicted by the Blasius equation defined by equation (16), but the liquid–wall friction factor did not follow the Blasius relation, but a correlation of the type

$$f_L = 0.263(R_L Re_{SL})^{-0.5}. \quad (17)$$

Kowalski [19] compared direct experimental measurements of the interfacial stress extrapolated from Reynolds shear profiles to indirect values determined from the momentum balance equation and showed a 13–20% deviation. Interfacial friction factor relationships were proposed for smooth stratified and stratified-wavy flow defined by equations (18) and (19), respectively,

$$f_i = 0.96 Re_{SG}^{-0.52} \quad (18)$$

$$f_i = 7.5 \times 10^{-5} R_L^{-0.25} Re_G^{-0.3} Re_L^{+0.83}. \quad (19)$$

Hart *et al.* [20] investigated stratified flows with small liquid hold-up values ($\bar{R}_L < 0.06$). A complex interfacial relationship was proposed which accounted for the distortion of the gas–liquid interface into a crescent-shaped film. Spedding and Hand [12] have shown excellent agreement existed between the Hart *et al.* [20] model and experiments ($< \pm 15\%$), with successful prediction of hold-up for the stratified and annular type flow regimes and for pressure drop prediction for the film plus droplet, annular wave and droplet and the droplet regimes.

In this investigation an extensive data bank compiled from pipelines with internal diameters between 0.025 m and 0.0953 m and systems with liquid viscosities between 0.001 N s m^{-2} and 0.1 N s m^{-2}, was used to evaluate model predictions. The full range of stratified flows were considered varying from a smooth gas–liquid interface to a wavy crescent-shaped interface approaching annular flow. Results showed the methodology used to calculate the liquid–wall and gas–liquid interfacial shear stresses were of importance in prediction of frictional pressure loss and hold-up in pipelines. Correlations for both the liquid–wall and interfacial friction factors were proposed which, when substituted into the one-dimensional momentum balance for stratified flow, gave accurate hold-up and pressure loss prediction.

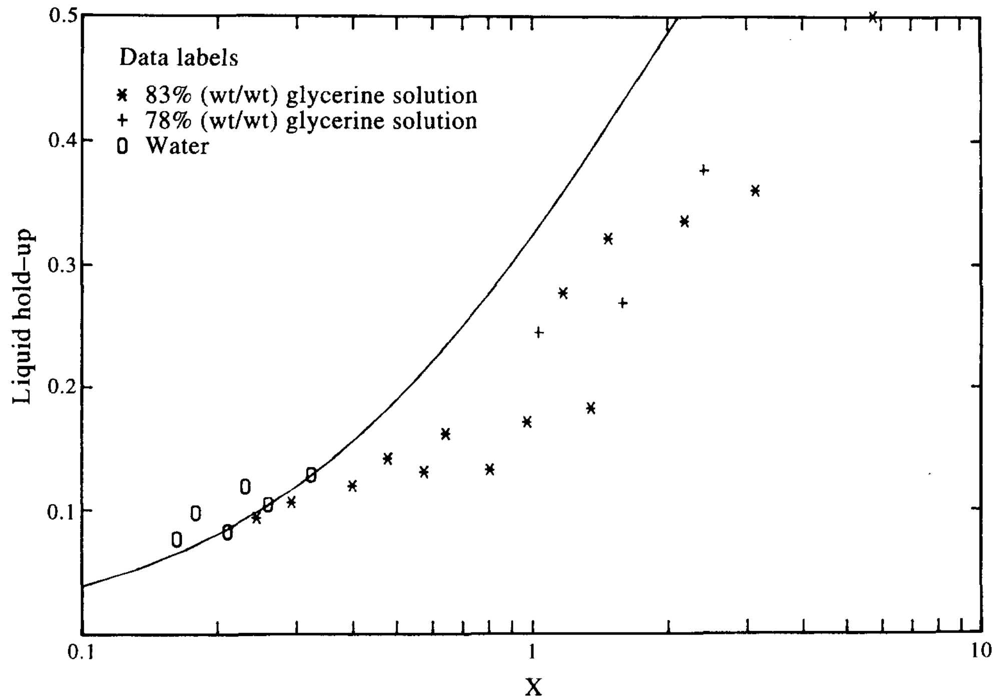

Fig. 3. R_L against log (X) with the smooth stratified flow data of Hand and Spedding [21], D = 0.0935 m, —— Taitel–Dukler [25] relation.

2. EXPERIMENTAL DATA BANK COMPOSITION

Hold-up and pressure loss data for two-phase co-current air–liquid flow in horizontal pipelines were utilised in evaluating stratified flow models. The data were obtained on 0.0935 m i.d. pipe by Hand and Spedding [21], on 0.0508 m i.d. pipe by Spedding and Ferguson [22], on 0.0935 m and 0.02515 m i.d. pipe by Andritsos [18] and on 0.0454 m i.d. pipe by Nguyen [23].

3. NON-UNIFORM STRATIFIED FLOW

An interfacial level gradient (ILG) can be present on stratified flows in horizontal pipelines at low gas velocities when the gas moves independently above the liquid. Such non-uniform flow can affect liquid hold-up measurement and flow pattern transitions, and can incur error because holdup varies along the pipeline. Bishop and Deshpande [24] have shown that the effect of ILG was magnified with either increased liquid viscosity or pipe diameter. In stratified flow experiments they recommended that pressure loss should be measured in each phase because the ratio $(\mathrm{d}P/\mathrm{d}L)_L/(\mathrm{d}P/\mathrm{d}L)_G$ was a quantitative measure of ILG. They proposed that a smooth stratified flow became uniform on satisfying the phase momentum, balance equations or the non-dimensional Taitel and Dukler [25] equation. Figure 3 shows the smooth stratified flow hold-up data of Hand and Spedding [21] for air–water, air–78% (wt/wt) glycerine and air–83% (wt/wt) glycerine systems plotted against the dimensionless Lockhart–Martinelli [26] parameter X, for liquid–gas laminar–turbulent flow. The extent of deviation between experimental hold-up and the Taitel and Dukler [25] prediction was a quantitative measure of ILG. In agreement with visual observations, viscous glycerine solutions exhibited significant ILG. Figure 4 shows the interfacial level profile measured by Andritsos [13] along a pipeline using a series of conductance probes. Under non-uniform flow conditions liquid hold-up was insensitive to changes in gas velocity and dependent solely on the location along the pipeline length. Increased gas velocities acted initially to depress the ILG rather than reduce the liquid height until uniform flow was obtained. Consequently the area of the stratified flow regime was expanded in a flow regime map similar to stratified flows in slightly downwardly inclined pipelines, until uniform flow was obtained.

4. ESTIMATION OF THE GAS-WALL SHEAR STRESS (τ_{WG}) AND LIQUID-WALL SHEAR STRESS (τ_{WL})

The experimental single phase gas wall friction factor data measurements were in all cases consistent with the turbulent Blasius equation (16). This was in agreement with other investigators such as Taitel and Dukler [25]; Andreussi and Presen [27] and Kowalski [19]. However, liquid–wall friction data were in error with this type of approach. Values of the liquid–wall shear stress were estimated from measurements of liquid hold-up and pressure loss using equation (6) by assuming f_G was accurately predicted from equation

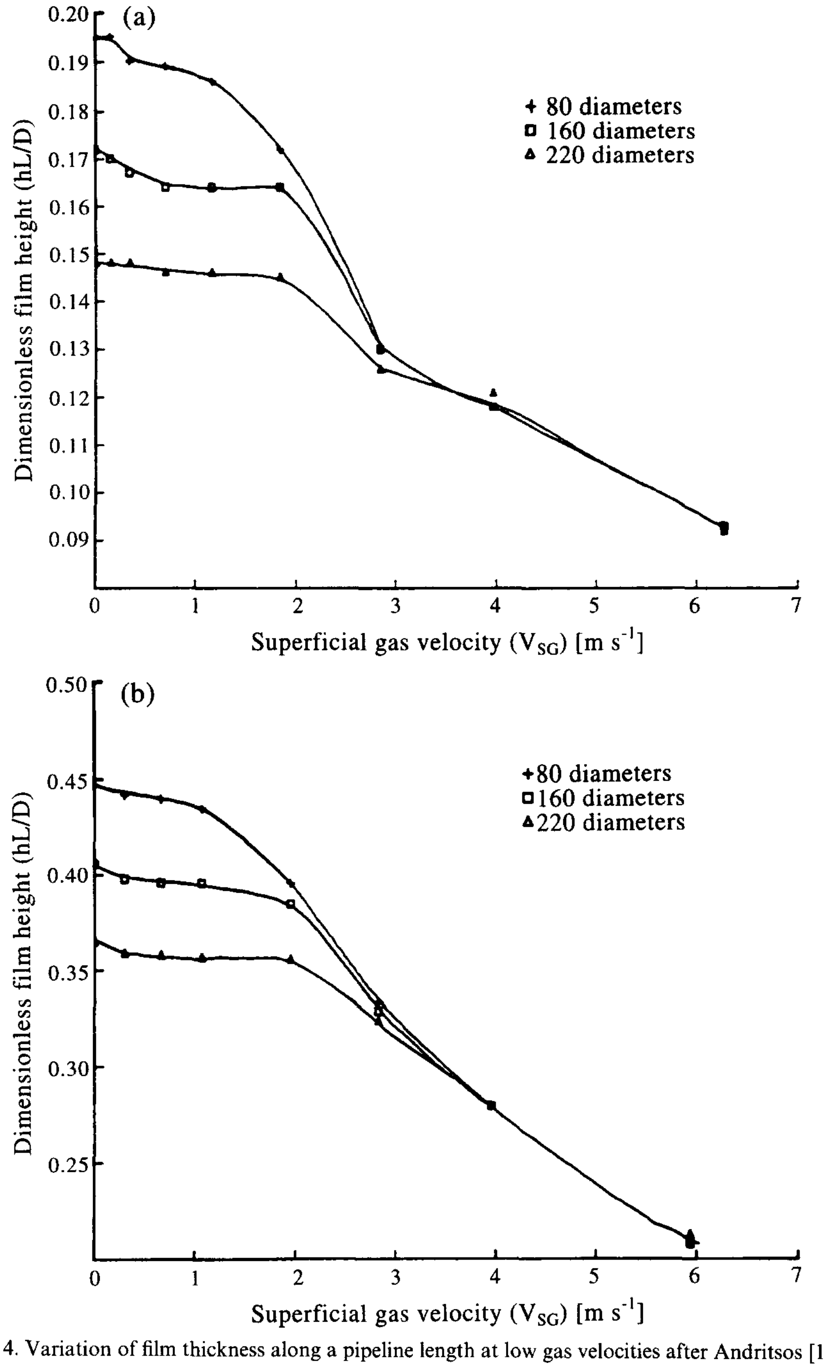

Fig. 4. Variation of film thickness along a pipeline length at low gas velocities after Andritsos [18].

(16) for turbulent gas flow. Non-uniform stratified data flows with an interfacial level gradient were excluded from the analysis because $(dP/dL)_L \neq (dP/dL)_G$. Uniform liquid gas laminar–turbulent stratified data were greater than equation (15) by on average 25%, as shown in Fig. 5. Equation (20) was found to be a better fit of the experimental data for both smooth and wavy interfacial conditions when $Re_L < 2100$.

$$f_L = 24Re_L^{-1}. \qquad (20)$$

An example of the values of f_L determined for turbulent–turbulent flow are shown in Fig. 6. The Blasius equation underestimated f_L for wavy stratified patterns. Deviation between the data and the Blasius prediction increased with increasing gas velocity. The same phenomenon was described by Andreussi and Persen [27], Andritsos [18] and Kowalski [19]. However, Andreussi and Persen [27] reported that the deviation reached a maximum at the onset of roll waves. The latter was in obvious disagreement with Fig. 6.

The above phenomenon demonstrated a definite effect of gas drag on the liquid velocity. Momentum

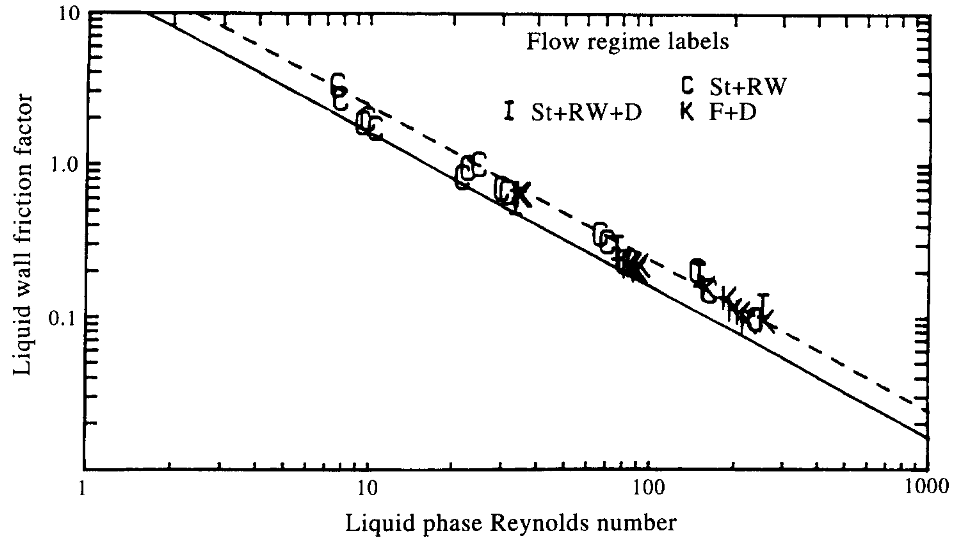

Fig. 5. f_L against Re_L, with air 83% (wt/wt) glycerine solution data of Hand and Spedding [21], laminar liquid, turbulent gas flow. --- equation (20), —— equation (15).

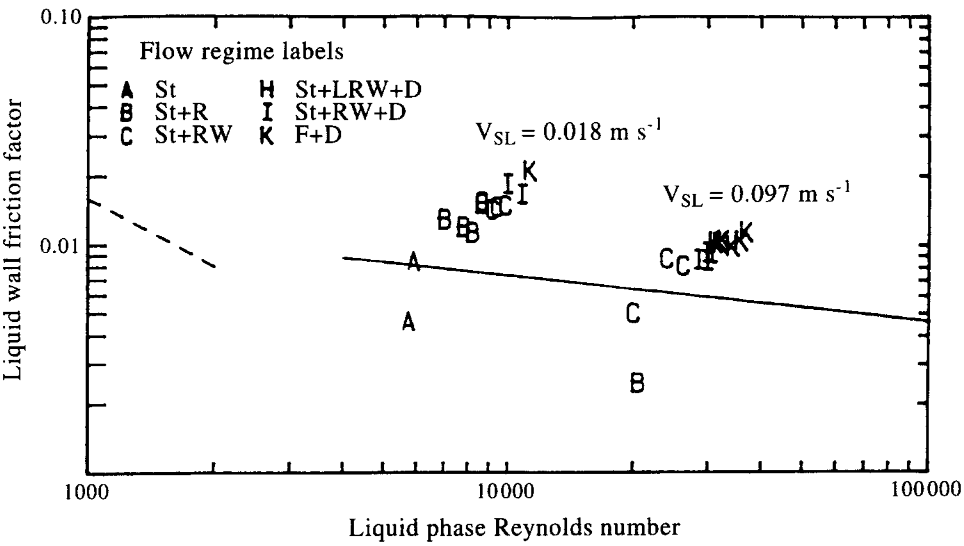

Fig. 6. f_L against Re_L, for air water data of Hand and Spedding [21], for $V_{SL} = 0.018$ m s^{-1} and $V_{SL} = 0.097$ m s^{-1}. --- equation (15), —— equation (16).

exchange across the gas–liquid interface increased the liquid velocity and consequently f_i.

Andritsos [18] proposed an alternative approach for calculating τ_{WL} based on the analysis of Cheremisinoff and Davis [15] (cf. equations (10)–(12)). However Fig. 7 shows an example of the inaccuracy of the method.

A better approach was presented by Kowalski [19], as shown by Fig. 8. The scatter of the data was greatly reduced by including the product of liquid hold-up R_L and Re_{SL} as the abscissa. This modified Reynolds number indirectly accounted for gas drag at the interface because hold-up varied with gas velocity. However, the equation suggested by Kowalski [19] did not compare well with the air–water data of Andritsos [18] and Hand and Spedding [21] and a more accurate correlation was developed as defined by equation (21).

$$f_L = 0.0262(R_L\, Re_{SL})^{-0.139}. \quad (21)$$

5. ESTIMATION OF THE INTERFACIAL FRICTION FACTOR

The apparent interfacial friction factor was estimated using the experimental measurements of pressure gradient and hold-up, and evaluated against the various relationships proposed in the literature. Most of the suggested models performed badly. For example the Ellis and Gay [14] relationship based on the single phase gas Reynolds number grossly underestimated f_i, as illustrated in Fig. 9. An improvement

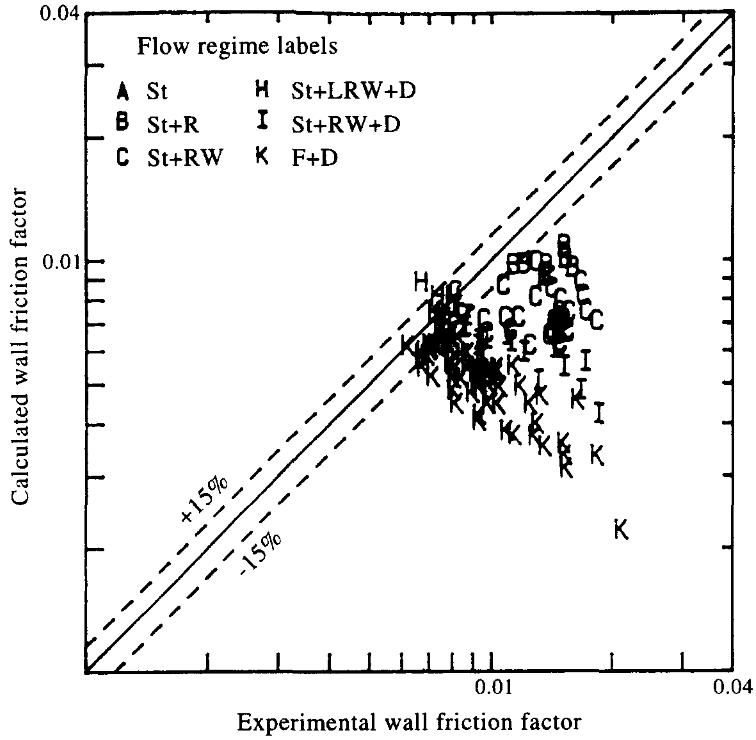

Fig. 7. Comparison between f_i determined using Andritsos [18] analysis and experimental f_L with Hand and Spedding [21] uniform turbulent–turbulent air–water data.

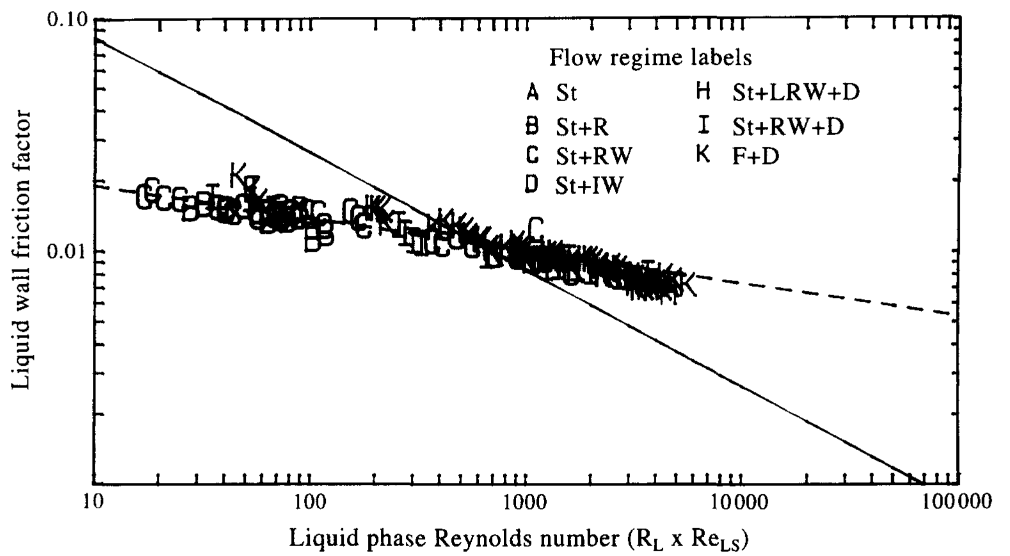

Fig. 8. f_L against ($R_L \cdot Re_{SL}$) with uniform air water data of Hand and Spedding [21]. —— equation (17), --- equation (21).

in the quality of prediction was achieved by employing the single phase liquid Reynolds number as the correlating parameter as suggested by Cohen and Hanratty [16] and Miya *et al.* [17] (cf. equations (8) and (9)) as shown in Fig. 10. Kowalski [19] realised that the relation was better served by including the effect of both phase Reynolds numbers in the form of equation (19) for stratified wavy flow. Figure 11 shows a substantial improvement in prediction performance. However, room for improvement still existed.

In Fig. 12 a dimensionless plot is given of f_i/f_{SG} against $V_{SG}/6$. The numeric 6 possessed dimensions of m s^{-1} and represented the reference superficial velocity at which the transition to stratified and roll wave flow pattern occurred and ILG became unsustainable. At ratios less than one the data showed scatter because

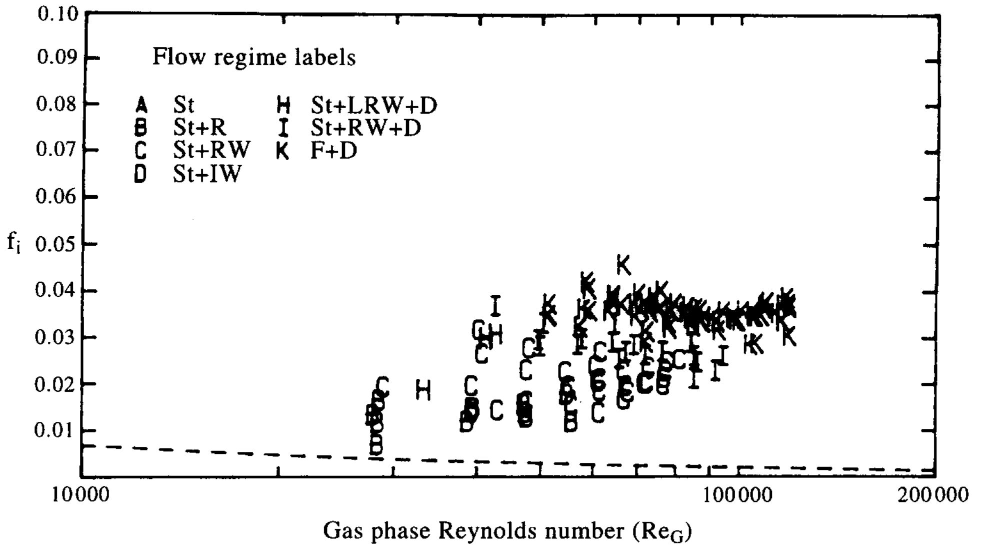

Fig. 9. f_i against Re_G with the uniform air water data of Hand and Spedding [21]. --- equation (7).

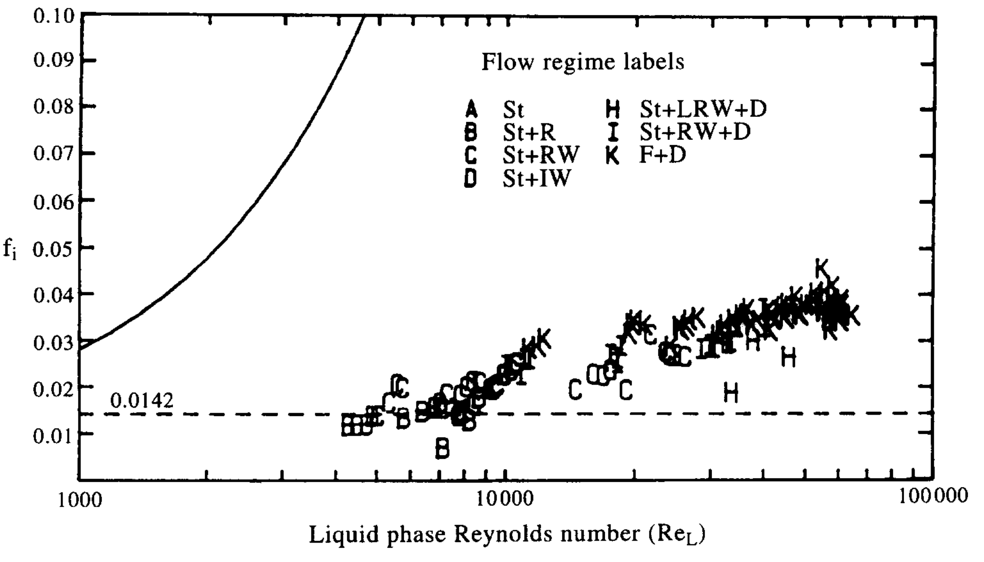

Fig. 10. f_i against Re_L with the uniform air water data of Hand and Spedding [21]. —— equation (9), --- equation (8).

flow was non-uniform. At the onset of ripples f_i/f_{SG} increased because interfacial roughness became greater. Furthermore initiation of large amplitude roll waves further augmented the roughness on the interface and the relationship became linear and was correlated by:

$$\frac{f_i}{f_{SG}} = 1.76\left(\frac{V_{SG}}{6}\right)+k_i. \tag{22}$$

The constant k_i varied with V_{SL} as shown in Fig. 13 plotted against $(\beta_L)_r$ (where $(\beta_L)_r$ represented a reference volumetric fraction calculated at the reference gas superficial velocity of 6 m s^{-1}). The reaction was correlated by

$$k_i = 2.7847\log_{10}(\beta_L)_r+7.8035 \tag{23}$$

$$(\beta_L)_r = \frac{V_{SL}}{V_{SL}+6}. \tag{24}$$

Equations (22)–(24) allow the interfacial friction to be calculated independently of liquid hold-up and pipe diameter from input flow rates. Figure 14 shows that the proposed relationship accurately predicted interfacial friction for all the uniform air water data. However, the linear relationship described by equation (24) was exclusively for air–water turbulent–turbulent flow. Results showed that the laminar–turbulent glycerine air data varied randomly suggesting the viscous glycerine laminar film dissipated gas drag differently from the turbulent water film at comparable liquid film heights and gas rates.

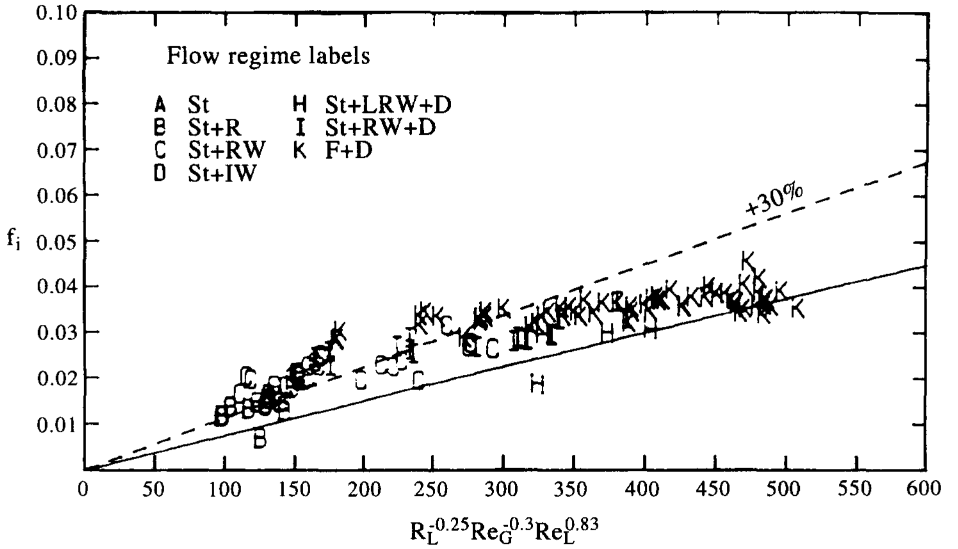

Fig. 11. Kowalski [19] friction factor plot with air water data of Hand and Spedding [21]. —— equation (19).

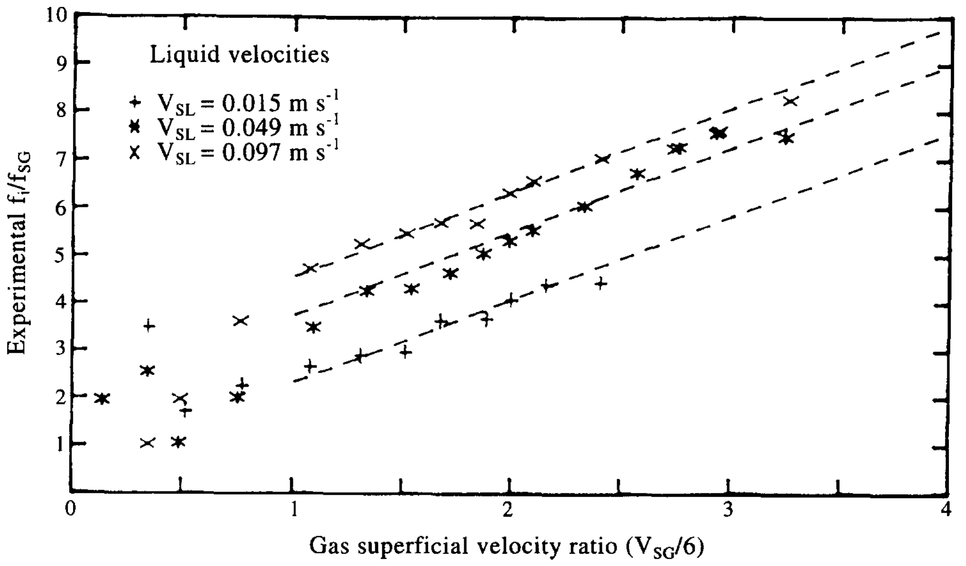

Fig. 12. f_i/f_{SG} against $V_{SG}/6$ with the uniform air–water data of Hand and Spedding [21]. --- equation (22).

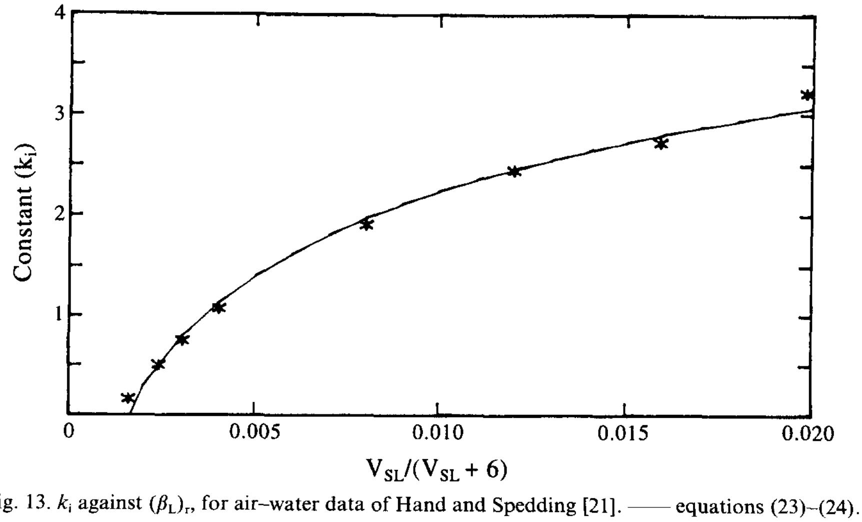

Fig. 13. k_i against $(\beta_L)_r$, for air–water data of Hand and Spedding [21]. —— equations (23)–(24).

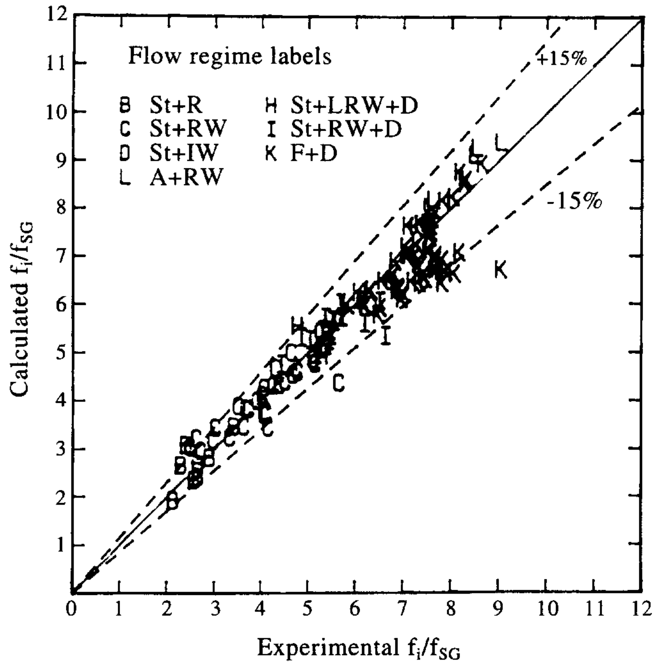

Fig. 14. Comparison between f_i/f_{SG} calculated using equation (22) and experimental values, with uniform air–water data of Hand and Spedding [21].

6. HOLD-UP AND PRESSURE LOSS PREDICTION

The prediction performance of the Andritsos [18] variation of the momentum balance model for liquid holdup and pressure loss estimation was tested against data. The model gave poor prediction of liquid hold-up but a better result for pressure loss estimation. In Fig. 7 it was shown that the method used by Andritsos [18] to calculate the liquid–wall shear stress τ_{WL} was inaccurate.

However, by substituting equation (21) in the calculation the prediction of hold-up was improved considerably. However the pressure loss was then overestimated, which was in agreement with the performance of the Andritsos [18] friction factor relationship as found in this work.

These results prove that the prediction of hold-up and pressure loss is sensitive to the methods used to determine the liquid–wall and interfacial friction factors.

7. A NEW PROCEDURE TO DETERMINE LIQUID HOLD-UP AND PRESSURE LOSS

Incorporating the new relationships for f_L and f_i from equations (20)–(24), the following iterative procedure is recommended for pressure loss and hold-up prediction for stratified type flow patterns. The flow rates, fluid properties and pipe diameter are required as input variables.

(a) Assume a value of h_L/D $(0 < h_L/D < 1)$ then calculate the variables A_L, A_G, S_L, S_G, S_i, and liquid hold-up from the geometrical considerations detailed in the Appendix. Subsequently evaluate the phase Reynolds numbers from the input volumetric flow rates.

(b) Determine phase-wall and superficial gas–wall friction factors using the following equations:

if $Re_L < 2100$,

$$f_L = 24Re_L^{-1}$$

if $Re_L > 2100$,†

$$f_L = 0.0262(R_L\, Re_{SL})^{-0.139}$$

if $Re < 2100$,

$$f_G = 16Re_G^{-1}$$

$$f_{SG} = 16Re_{SG}^{-1}$$

if $Re_G > 2100$,†

$$f_G = 0.046Re_G^{-0.2}$$

$$f_{SG} = 0.046Re_{SG}^{-0.2}.$$

(c) Calculate the interfacial friction factor using the following relationships:

(i) For air–water systems use equations (22)–(24) to determine f_i/f_{SG}

(ii) For air–viscous liquid systems the Andritsos [18] method of equations (13) and (14) is recommended where $(V_{SG})_t = 5\ \text{m s}^{-1}$ at atmospheric pressure.

† Note turbulent flow is not fully developed until Re_G, $Re_L > 4000$ [9].

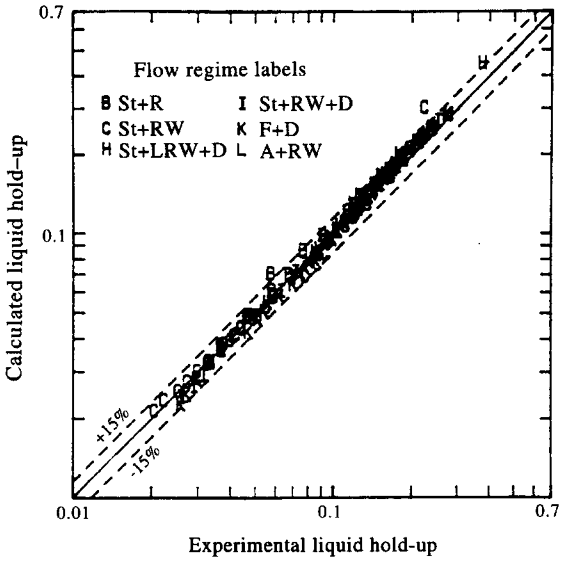

Fig. 15. Comparison between $\bar{R}_L$ predicted using new procedure and experimental values, with the air–water data of Hand and Spedding [21].

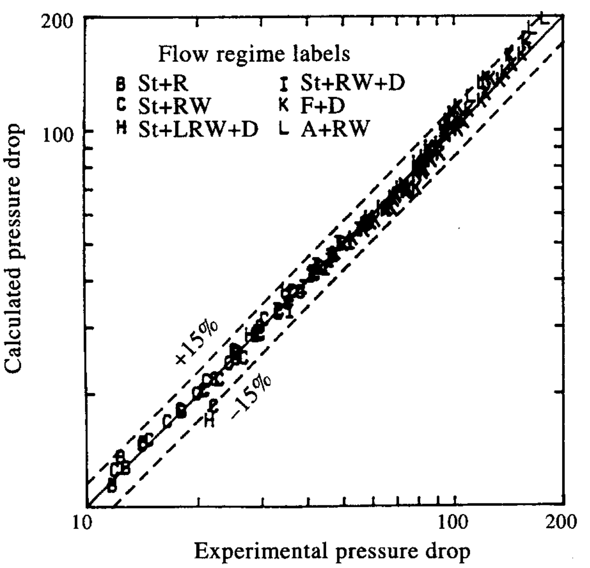

Fig. 16. Comparison between pressure loss (N m m^{-1}) predicted using new procedure and experimental values, with the air–water data of Hand and Spedding [21].

(d) Determine phase-wall and interfacial shear stress using equations (3)–(5).

(e) Calculate phase pressure drops in both phases using equations (1) and (2), respectively.

(f) Compare the calculated phase pressure drops. If they do not agree within a tolerance of 0.0001 N m^{-1}, assume a new value of h_L/D and recalculate again at step (1).

8. EVALUATION OF NEW MODEL AGAINST DATA BANK

Predicted liquid hold-up and pressure loss values calculated from the suggested procedure are compared with the experimental air–water data of Hand and Spedding [21] for a 0.0935 m i.d. pipeline, in Figs. 15 and 16 respectively. The improvement over previous models can be attributed to the relationships for calculating f_L and f_i. Comparison with the independent air–water data of Andritsos [18] for 0.09525 and 0.02515 m i.d. pipelines and Nguyen [23] for a 0.0455 m pipeline and Spedding and Ferguson [22] for 0.0508 m pipeline also shows excellent agreement.

Similarly the laminar–turbulent liquid–gas air–83% (wt/wt) glycerine data of Hand and Spedding [21] compared accurately with theory as shown by Figs. 17 and 18.

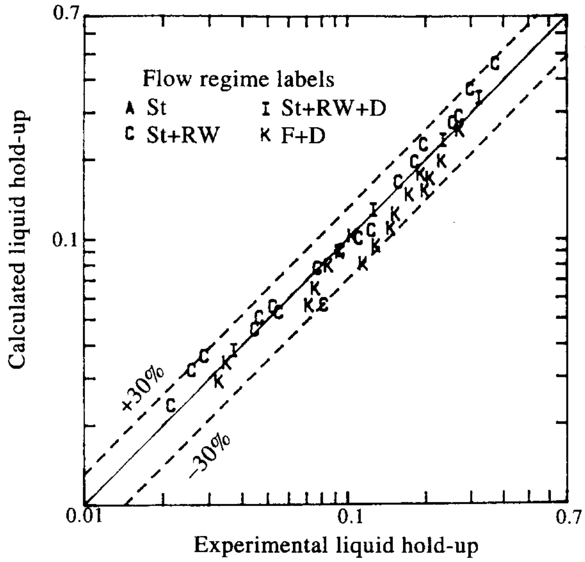

Fig. 17. Comparison between calculated hold-up $\bar{R}_L$ predicted using new procedure and experimental values, with air–83% (wt/wt) glycerine solution data of Hand and Spedding [21].

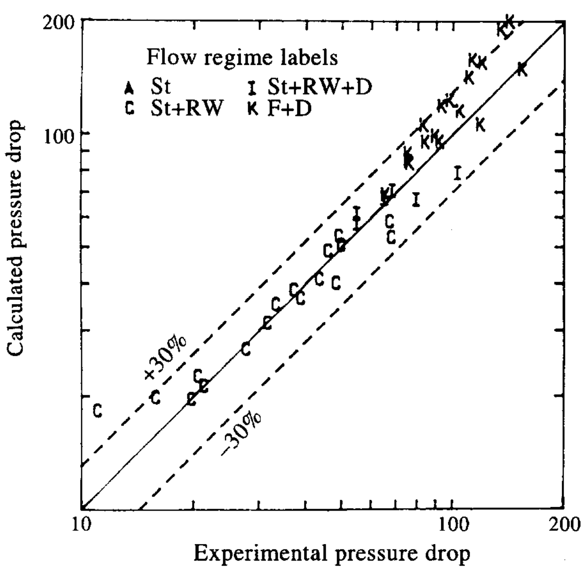

Fig. 18. Comparison between calculated pressure loss (N m m^{-1}) predicted using new procedure and experimental values, with air–83% (wt/wt) glycerine solution data of Hand and Spedding [21].

9. CONCLUSIONS

(a) At low gas non-uniform stratified flow with an interfacial level gradient (ILG) was observed visually along the pipeline length, particularly with the viscous glycerine solutions. The theoretical criteria of Bishop and Deshpande [24] gave good prediction of this phenomenon.

(b) Values of f_L and f_i evaluated indirectly from experimental hold-up and pressure loss measurements were compared with relationships from the literature. Values of f_L were found to be underestimated by the Poiseuille and Blasius equations for laminar and turbulent flow respectively. New relationships were proposed that gave more accurate prediction of f_L. Interfacial friction factors for air–water systems were correlated by a new relationship, equation (22).

(c) An iterative procedure incorporating the new relationship for f_L and f_i was proposed in the solution to the phase momentum balance equations (equations (1) and (2)). Evaluation against an extensive data bank comprised of 823 stratified observations verified the accuracy of the model.

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APPENDIX—GEOMETRIC PARAMETERS

For the stratified flow shown in Fig. 1, the following dimensionless parameters follow from geometry.

$$\hat{S} = \pi - \cos^{-1}(2\hat{h}_L - 1)$$

$$\hat{S}_G = \cos^{-1}(2\hat{h}_L - 1)$$

$$\hat{S}_i = \sqrt{1-(2\hat{h}_L - 1)^2}$$

$$\hat{A}_L = 0.25[\pi - \cos^{-1}(2\hat{h}_L - 1) + (2\hat{h}_L - 1)\sqrt{1-(2\hat{h}_L - 1)^2}]$$

$$\hat{A}_G = 0.25[\cos^{-1}(2\hat{h}_L - 1) + (2\hat{h}_L - 1)\sqrt{1-(2\hat{h}_L - 1)^2}]$$

$$\hat{A} = \frac{\pi}{4}$$

$$S_{L,G,i} = \hat{S}_{L,G,i} \cdot D$$

$$A_{L,G} = \hat{A}_{L,G} \cdot D^2$$

$$h_L = \hat{h} \cdot D$$

$$\bar{R}_G = 1 - \bar{R}_L = \frac{1}{\pi}[\cos^{-1}(2\hat{h}_L - 1) - (2\hat{h}_L - 1)\sqrt{1-(2\hat{h}_L - 1)^2}]$$

$$\bar{V}_{L,G} = \frac{\bar{V}_{SL,SG}}{\bar{R}_{L,G}}$$

$$D_L = \frac{4A_L}{S_L}$$

$$D_G = \frac{4A_G}{S_G + S_i}$$

$$Re_L = D_L \bar{V}_G \rho_L / \mu_L$$

$$Re_G = D_G V_G \rho_G / \mu_G.$$